AN EFFECT WHICH STABILIZES THE CURVED FRONT OF A LAMINAR FLAME

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Investigation of the hydrodynamic stability of the plane front of a laminar flame was started in 1944 by L. D. Landau [1], who obtained the paradoxical result of absolute instability of the plane front in the linear approximation. In Landau's work, the laminar flame was represented in the form of a surface of discontinuity of temperature, pressure, density, and velocity which propagated relative to the gas at a constant velocity not dependent on the curvature of the surface.

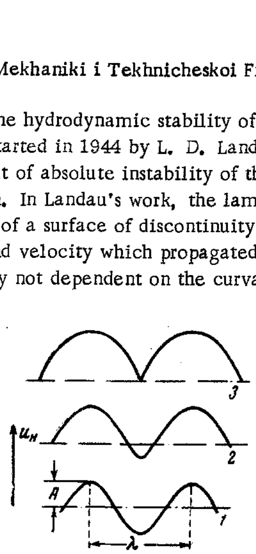

Fig. 1

Thus, the result obtained by Landau should be applied to those representations of a flame front where the wavelengths are large as compared with its thickness.

Curvatures of small wavelength influence the flame structure and change its velocity of propagation. The nature of this influence may vary according to the relationship between the thermal diffusivity and the diffusion coefficient limiting the chemical reaction rate (attention was called to this fact [2] even before the work of Landau). That is, if the thermal diffusivity exceeds the diffusion coefficient, then one might expect on the basis of physical considerations a decrease in the velocity of the flame in the convex sections and an increase in the concave ones, which would lead to stabilization of the flame. It is clear, however, that the Reynolds numbers for such small disturbances should be on the order of unity.

These qualitative considerations have been developed quantitatively in a number of references [3–5]. These sources imply that within the framework of linearized theory, the critical Reynolds number cannot exceed 10–20. This contradicts the experimental data—stable laminar flames have been observed at Reynolds numbers up to 10^4–10^6 in some experiments [6–8]. The solution of this paradox lies apparently, on the one hand, in peculiarities of the propagation of a flame under conditions in which the surface of the flame grows; A. G. Istratov and V. B. Librovich showed that in this case, the critical Reynolds number increased many times [9]. On the other hand, nonlinear effects are possible which might stabilize a flame if the disturbances on the surface of the flame were sufficiently large. Moreover, when considering flames in tubes and other devices, it is necessary to take the stabilizing effect of the walls into account.

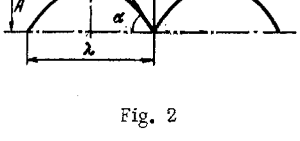

Fig. 2

In this communication we shall present ideas on a possible nonlinear effect which would ensure flame stabilization. Attention was directed to this effect for the first time in [10] and similar considerations were recently expressed by K. I. Shchelkin [11].

In Landau's theory, increases in disturbances in the propagation of a laminar flame occur as a result of non-one-dimensional motion of the gas connected with a pressure drop at the curved surface of the flame. We shall represent such an experiment theoretically. Suppose we shut off motion of the gas. Then, the hydrodynamic development of the disturbances would cease, the curved flame front propagating through the motionless gas at a constant normal velocity would begin to change shape, the convex sections of the flame would grow and the concave ones would diminish. Ultimately, corner points would appear in place of the convex sections. Figure 1 shows three successive positions of the flame front constructed by the Huygens method (circles with radius $u_H \Delta t$ are drawn about the points in the front, where u_H is the normal velocity of the flame, Δt is the time interval; then their envelope is drawn). Position 3 corresponds to the time of appearance of corner points on the flame.

Due to the merging of two sections of the front, the velocity of propagation of the corner points which have formed will be greater than the normal velocity of the flame. This leads to a situation in which the disturbances on the surface of the flame begin to decrease, as noted in reference [10]. Let us consider in a semiquantitative manner the stationary disturbed state to which an examination of this effect would lead. Let α be the angle of inclination of the flame front at a corner point (Fig. 2). Then the velocity of propagation of the corner point is equal to $u_H/\cos\alpha$. We have for the rate at which the amplitude of the disturbances decreases due to the high velocity of the corner points

$$\left(\frac{dA}{dt}\right)_- = -u_н\left(\frac{1}{\cos\alpha} - 1\right). \quad (1)$$

It is assumed here that the velocity of the flame at the corner point ceases to be equal to the normal velocity. Actually, due to the sharp curvature of the front at a corner point, the velocity of the flame will vary, and a region similar to the tip of a Bunsen flame will appear at the corner point. By assuming constant velocity, we thus neglect its structure.

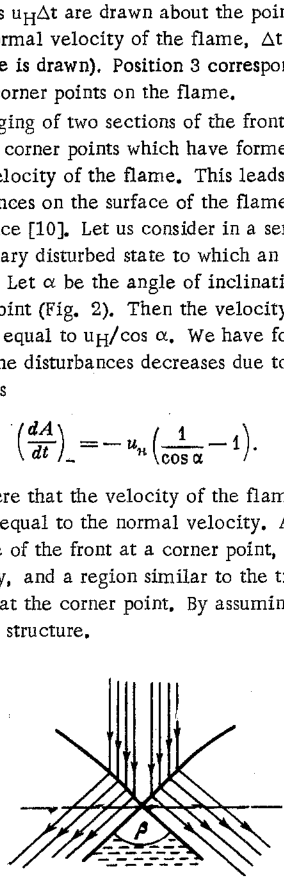

Fig. 3

Now, we shall give the shape of a curved flame. Let us assume that the flame front consists of sections of parabolas (Fig. 2). On the basis of geometric considerations, for small α we have

$$\operatorname{tg}\alpha = \alpha = \frac{4A}{\lambda} = \frac{2Ak}{\pi} \qquad \left(k = \frac{2\pi}{\lambda}\right). \quad (2)$$

Here, A is the amplitude, λ the wavelength of the disturbance, and k the wave number.

Therefore we rewrite (1) in the form

$$\left(\frac{dA}{dt}\right)_- = -\frac{2}{\pi^2} k^2 u_н A^2. \quad (3)$$

Thus, this effect of stabilization of a flame front by corner points is nonlinear and proportional to the square of the amplitude of the disturbance. This is clear because the effect (3) was not taken into consideration in Landau's linearized theory in which the Huygens principle was retained on condition that the normal velocity of the flame remained constant. We shall now establish the connection between Landau's theory and the effect considered here, which will permit us to find the relationship between the wavelength and the amplitude of disturbances for a stationary curved flame front.

The following relationship holds in the linear approximation:

$$\frac{dA}{dt} = \omega A, \qquad \omega > 0 , \tag{4}$$

where ω is the characteristic frequency of the problem.

Considering (4) as the first term of the expansion of the derivative dA/dt in a series in powers of A, we write this expression with accuracy to the next term:

$$\frac{dA}{dt} = \omega A + \varkappa A^2. \tag{5}$$

Here $\varkappa$ is a certain constant. We find from considerations of dimensionality that

$$\omega = \Omega k u_{н}, \qquad \varkappa = \mu k^2 u_{н} , \tag{6}$$

where Ω and μ are dimensionless constants.*

It can be seen from (5) that if $\mu < 0$, a disturbed front will not be agitated indefinitely, but will reach a stationary curved state with the amplitude

$$A_* = -\Omega/\mu k. \tag{7}$$

One may evaluate μ by comparing formulas (5) and (3):

$$\mu = -2/\pi^2. \tag{8}$$

This method of determining μ is approximate, since all second-order terms in the equations are not taken into consideration consistently. However, this method apparently satisfactorily describes the principal stabilizing effect connected with the presence of corner points on the flame. To obtain an exact determination of μ, it is necessary to solve the nonlinear problem of the motion of a gas when a curved flame is propagated, which still involves great difficulties.

The ideas set forth here give the relationship between the wavelength and the amplitude of the disturbance, but they do not determine the wavelength itself on a stationary curved flame. An additional condition is essential for its determination. One may assume, for example, that the wavelength corresponds to those initial disturbances whose rate of growth is maximum (refer to reference [2] in which the wavelength of a disturbance growing at a maximum rate is calculated).

*L. D. Landau utilized the same procedure in the theory of the turbulence. Here, unlike the case of turbulence, the motion does not come from a self-oscillating mode, but from a stationary mode. Thus, the expansion (5) contains quadratic terms which drop out in the theory of turbulence due to averaging over periods of time which are large compared with the period of oscillation of the amplitude [12].

We also call attention to the fact that corner points on a flame front lead to the formation of stagnant regions in combustion products. This can be seen from Fig. 3, in which gas streamlines close to a corner point are shown. Refraction of streamlines caused by thermal expansion of gas leads to a situation in which not a single streamline may fall within the region behind a corner point—thus this region should be filled by gas which is not moving relative to the flame front. It is easy to find the angle β in this region:

$$\beta = 2\alpha(1-r) = \frac{4Ak}{\pi}(1-r). \tag{9}$$

Here r is the ratio of the density of combustion products to the density of the cold gas.

The action of viscosity, and also the turbulent diffusion of the tangential discontinuity forming behind a corner point, leads to a situation in which a stagnant region exists only close to a corner point. A turbulent wake occurs behind a corner point at great distances from the flame front.

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